Hilroy

80 Pages
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EXERCISE BOOK
CAHIER D'EXERCICES

NAME/NOM ______________________

SUBJECT/SUJET ELEC 341

ASSEMBLED IN CANADA WITH IMPORTED MATERIALS
ASSEMBLÉ AU CANADA AVEC DES MATIÈRES IMPORTÉES

12107

Laplace Transform

5/07/19
LEC

Laplace Transform Properties

① Time Delay, $f(t-T)\,u(t-T) \overset{L}{\longleftrightarrow} e^{-Ts}F(s)$

② Differentiation, $f'(t) \overset{L}{\longleftrightarrow} sF(s) - f(0)$

③ Integration, $\int f(t)\,dt \overset{L}{\longleftrightarrow} \frac{F(s)}{s}$

SS 14

④ Final Value Thrm, "open" does not include $j\omega$ axis
"closed" includes $j\omega$ axis

SS 15

"If all poles of $sF(s)$ are in "open" LHP plus maybe a simple pole at the origin"

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Simple: "s in the denominator to the power 1"

SS 16

⑤ Initial Value Thrm,

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$$; if the limit exists

"doesn't matter where the poles are located"

SS 18

⑥ Frequency shift, $e^{-at}f(t) \overset{L}{\longleftrightarrow} F(s+a)$

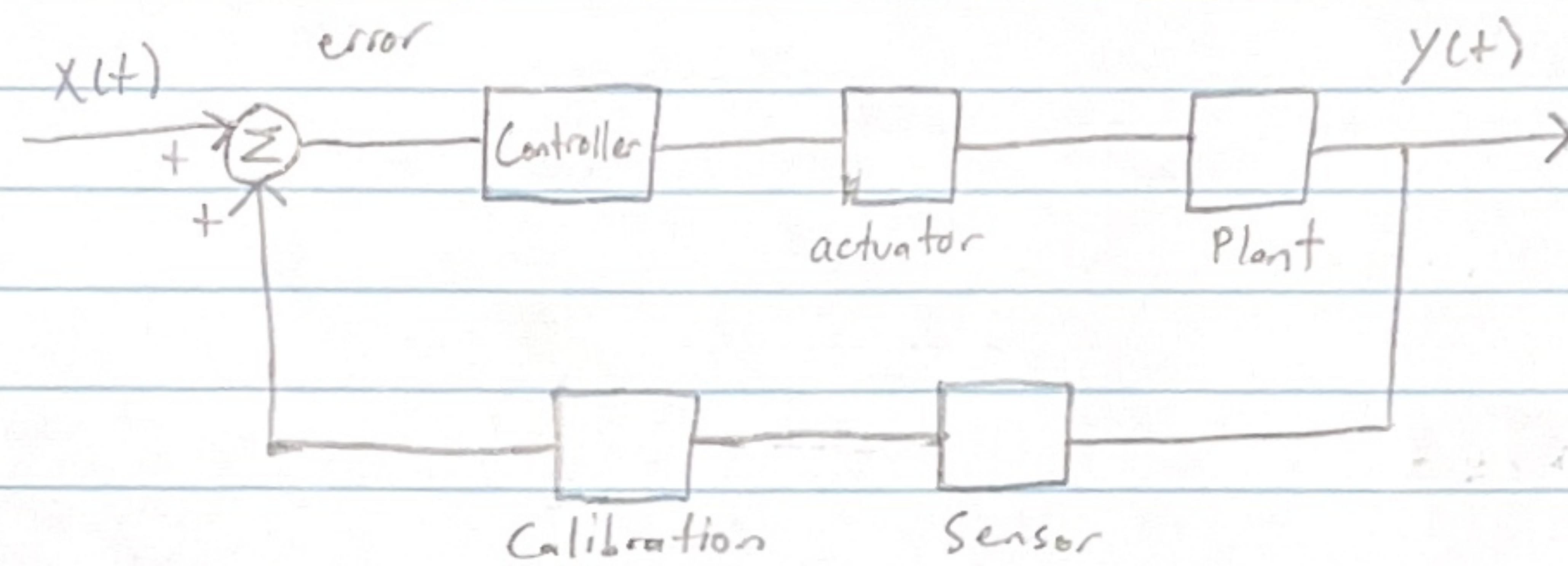
Useful: $L\{t^k f(t)\} = (-1)^k \frac{d}{ds^k}[F(s)]$

Modeling

5/08/19
LEC

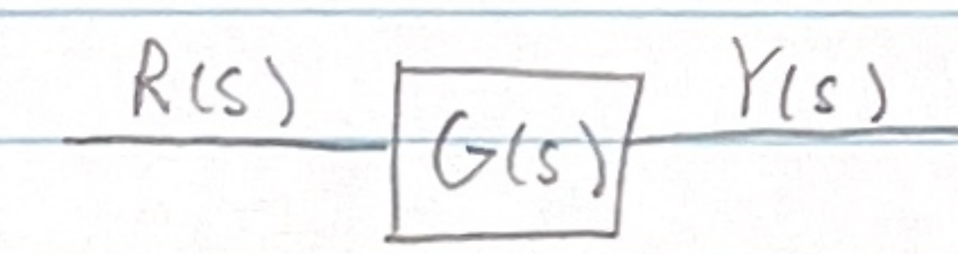
$\boxed{SS\ 2}$ Pressure Tranducer, $p = \rho g h$

$\boxed{SS\ 3}$



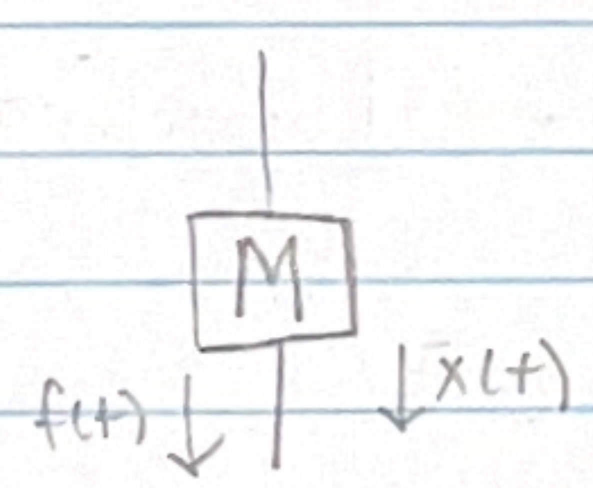
$\boxed{SS\ 5}$ Deadband, tolerance

$\boxed{SS\ 12}$ Transfer Function,



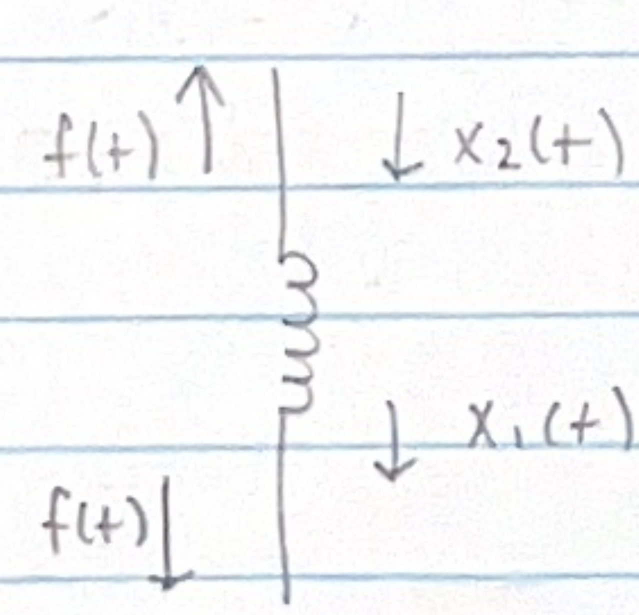
$$G(s) = \frac{Y(s)}{R(s)}$$

$\boxed{SS\ 24}$ ① Mass,



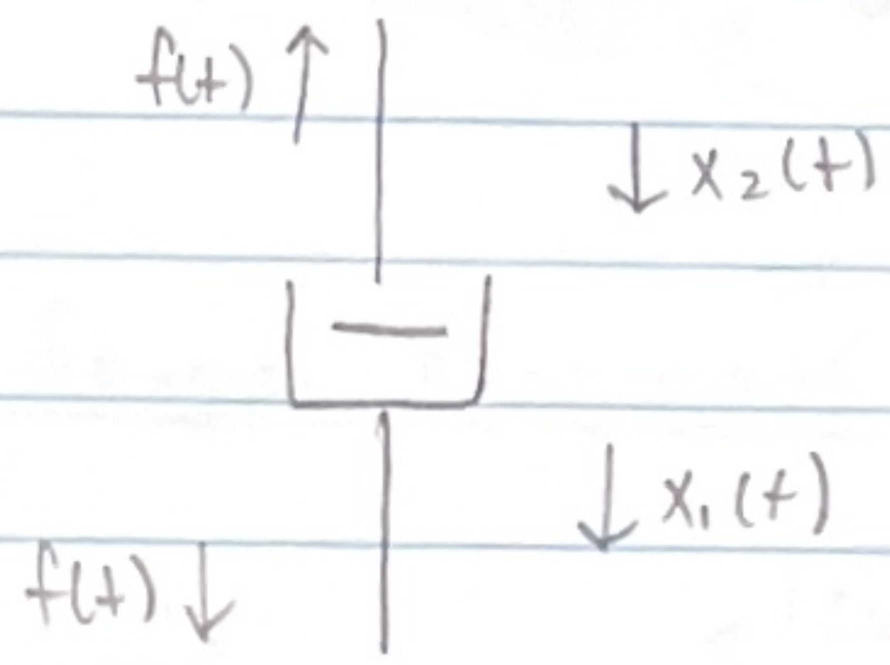
$$f(t) = M x''(t)$$
$$F(s) = M s^2 X(s)$$

② Spring,



$$f(t) = k(x_1(t) - x_2(t))$$
$$F(s) = k(X_1(s) - X_2(s))$$

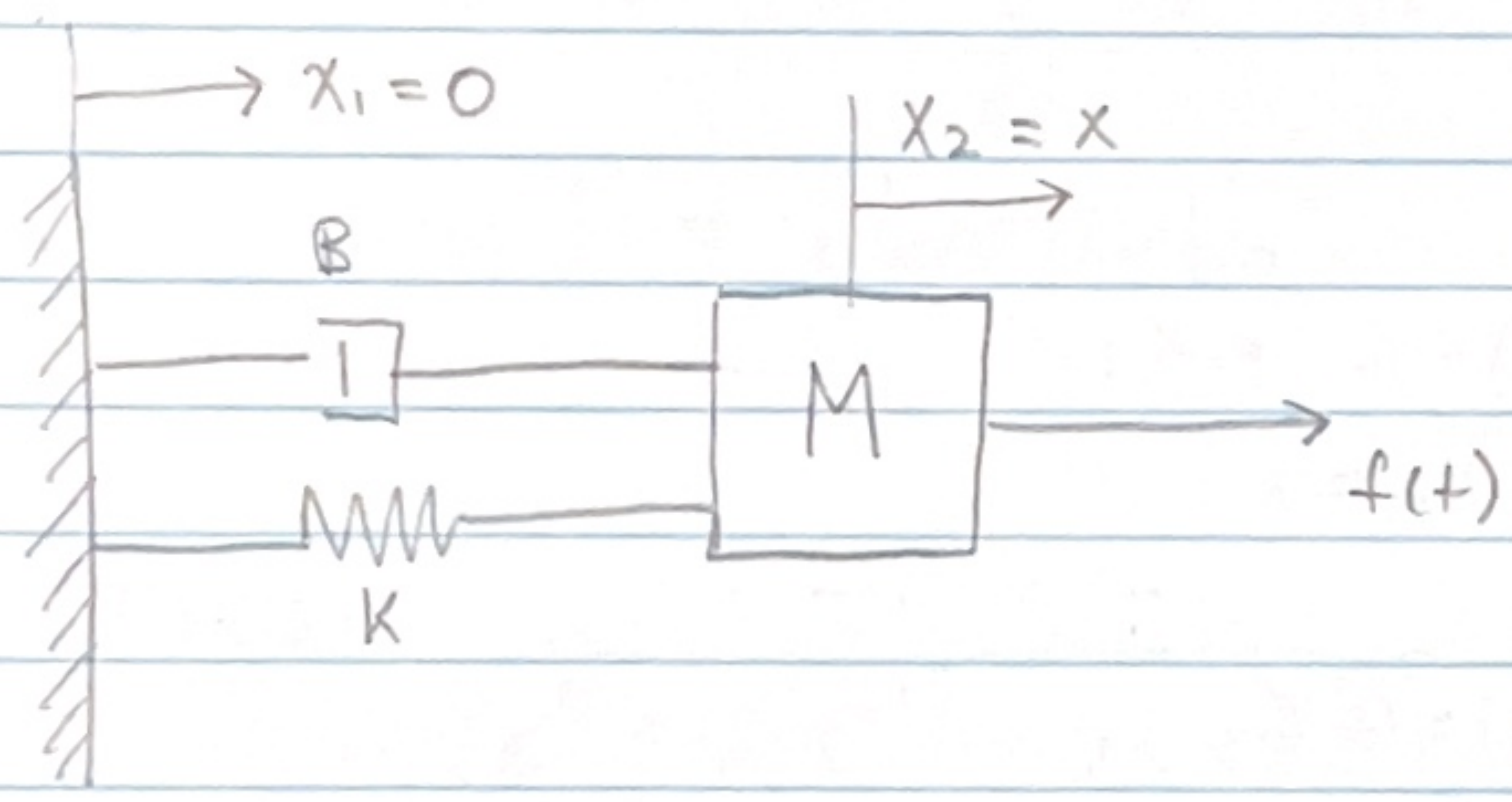
③ Damper



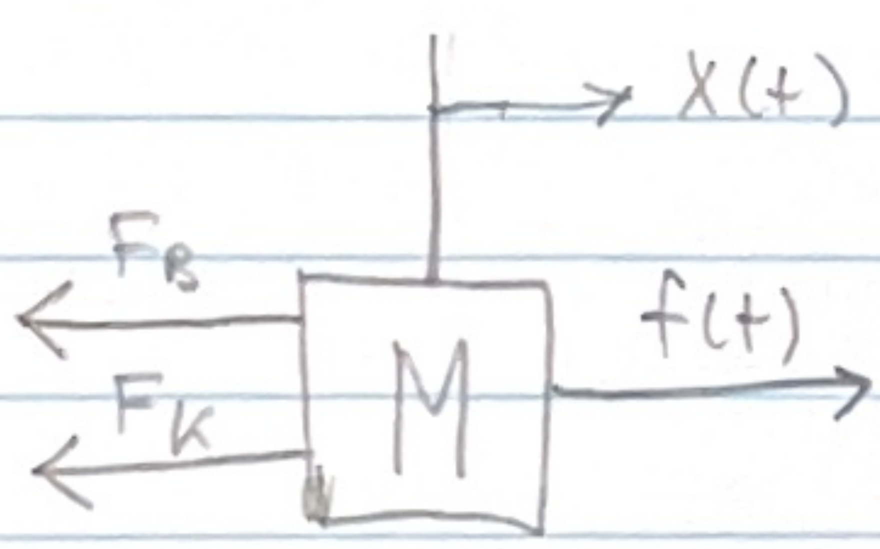
$$f(t) = B(x_1'(t) - x_2'(t))$$
$$F(s) = Bs(X_1(s) - X_2(s))$$

Similarities

Mass ~ Capacitor
Spring ~ inductor
Damper ~ Resistor



FBD



$$M\ddot{X}_2 = f(t) - F_B - F_K$$

$$M\ddot{X} = f(t) - B\dot{X} - kX$$

$$F_B = (-B)(\dot{X}_{LEFT} - \dot{X}_{RIGHT})$$
$$= (-B)(0 - \dot{X})$$
$$= B\dot{X}$$

$$F_K = (-k)(X_{LEFT} - X_{RIGHT})$$
$$= (-k)(0 - X)$$
$$= kX$$

Modeling Contd.

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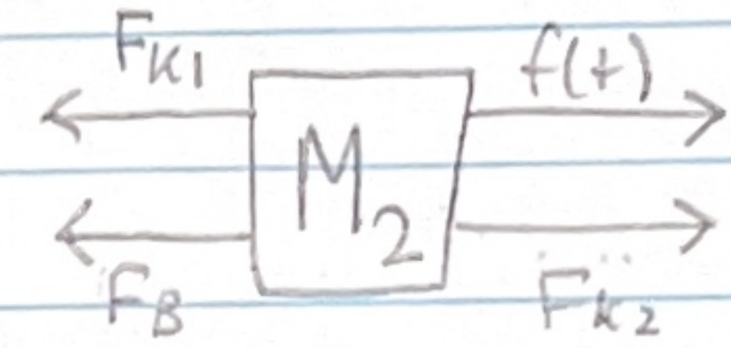
SS 26

FBD



$$M_2 \ddot{x}_2 = -F_{k1} - F_B + F_{k2} + f(t)$$
$$= [(-k_1)(x_1 - x_2)] - [(-B)(\dot{x}_1 - \dot{x}_2)] + [(-k_2)(x_2 - \cancel{x_3}^{\,0})] + f(t)$$

$$M_2 \ddot{x}_2 = f(t) - B(\dot{x}_2 - \dot{x}_1) - k_1(x_2 - x_1) - k_2 x_2$$

SS 28

$$X_2 G_1 = X_1$$

$$(F \cdot G_2 + X_1 \cdot G_3) \cdot G_1 = X_1$$
$$F G_1 G_2 + G_1 G_3 X_1 = X_1$$

$$F G_1 G_2 = X_1 (1 - G_1 G_3)$$

$$\frac{X_1}{F} = \frac{G_1 G_2}{1 - G_1 G_3}$$

SS 29

Similarities

1. Inertia (J) ~ Mass ~ Capacitor
2. Spring (K) ~ Spring ~ inductor
3. Friction (B) ~ Damper ~ Resistor

$\boxed{\text{State Space Representation} \longleftrightarrow \text{Transfer Function}}$

$$T(s) = \frac{Y(s)}{U(s)} = C(sI-A)^{-1} \cdot B + D$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} b_N - a_n b_0 & \ldots & b_1 - a_1 b_0 \end{bmatrix}$$

$$T(s) = \frac{}{s^N + a_1 s^{N-1} + a_2 s^{N-2} + a_3}$$

Lecture 5

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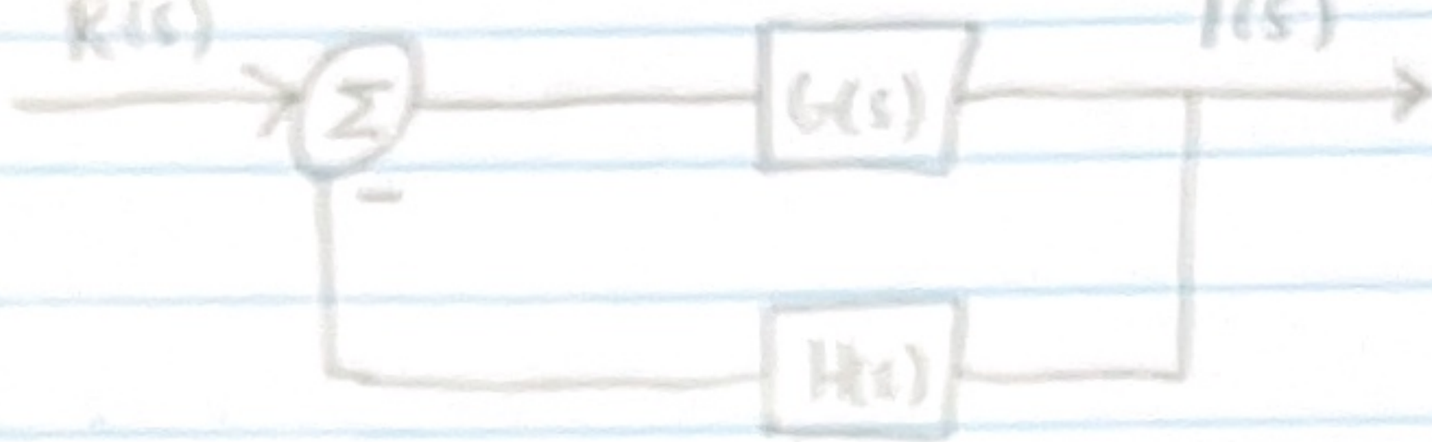
SS 11

Black's Formula:

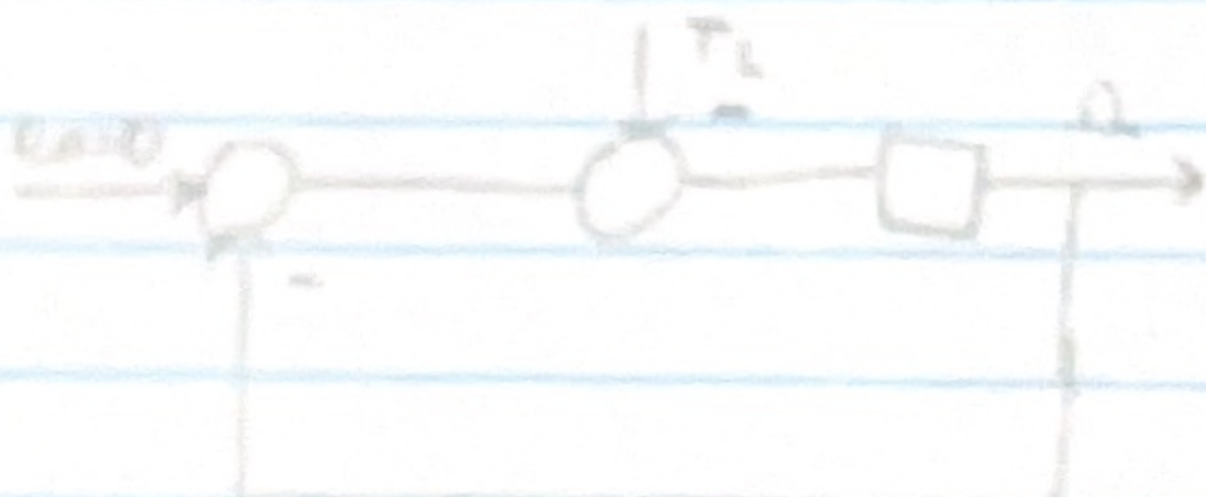


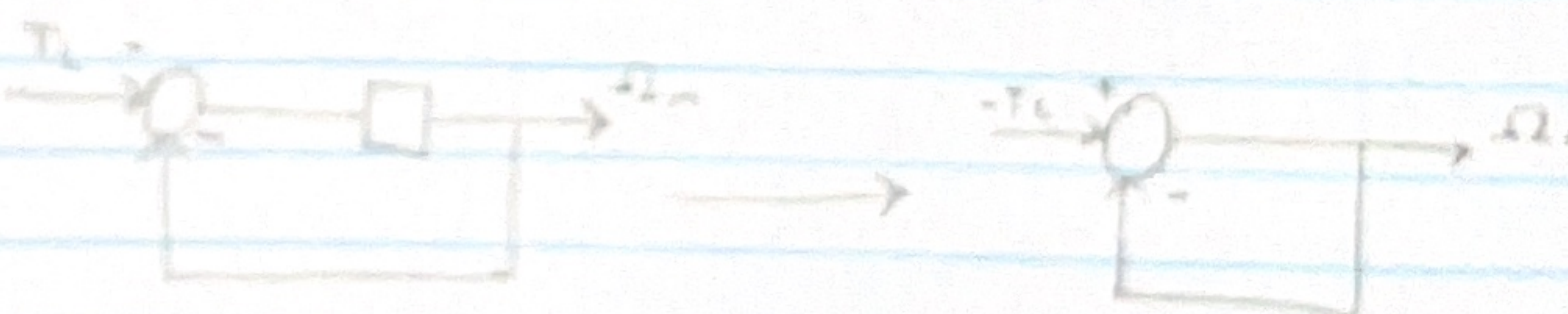
$$Y(s) = \frac{G(s)}{1 + G(s)H(s)} \cdot R(s)$$

*If $\boxed{\text{Positive}}$ feedback system, $Y(s) = \frac{G(s)}{1 - G(s)H(s)} \cdot R(s)$

SS 12

If $e_a = 0$:





$$\frac{\Omega_m}{-T_L} = \frac{\Omega}{1+\Omega}$$

$$\boxed{\frac{\Omega_m}{T_L} = -\frac{\Omega}{1+\Omega}}$$

* For multiple input systems, find individual input transfer functions then add via superposition

Standard Form:

$\boxed{1^{st} \text{ order}}$

$$H(s) = \frac{k}{Ts+1}$$

$\boxed{2^{nd} \text{ order}}$

$$H(s) = \frac{k}{s(Ts+1)}$$

"Generalized Method for Linearization"

Taylor Series Expansion:

$$g(x) \approx g(x_0) + \left.\frac{dg(x)}{dx}\right|_{x=x_0} (x-x_0)$$

Linearization Procedure:

1. Identify input and output variables
   $F(t) = \text{input}$
   $x(t) = \text{Output}$
2. Express non-linear ODE in the form $f(\ddot{x}, \dot{x}, x, F) = 0$
3. Find operating point x_0 and $F(x_0)$
4. Write the taylor series expansion at OP (x_0, F_0)
5. Change of variables
6. Re-write TSE as a linear ODE

Lecture 6: Stability

5/14/19
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SS 5

Types of Stability:

① BIBO $\longrightarrow \int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ 'or' $j\omega$ axis in ROC of $H(s)$
ie: Poles are in LHP

② Asymptotic Stability $\longrightarrow$ Poles are in LHP.

SS 11

Stability:

① STABLE $\rightarrow$ All poles on LHP
② poles on $j\omega$ axis $\longrightarrow$ 2 possibilities, ① Unstable ② Marginally stable
③ UNSTABLE $\rightarrow$ Poles on RHP

Marginally stable Criteria

① No poles on RHP
② At least 1 pole on $j\omega$ axis
③ all poles on $j\omega$ axis have multiplicity of 1

SS 14

$a = 1$
$b = B$
$c = k$

$$\frac{-B \pm \sqrt{B^2 - 4k}}{2} \rightarrow \frac{-B}{2} \pm \frac{\sqrt{B^2 - 4k}}{2}$$

$s^2 + 2s + 3$
$(s+1)(s+2)$

SS 15

① STABLE
② UNSTABLE
③ UNSTABLE
④ UNSTABLE

Lecture 6 Cont'd

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SS 17

Routh Hurwitz Criterion:

Ex: $5s^3 + 2s^2 + s + 5$

$b_1 = \frac{(1)(2) - (5)(5)}{2} =$

s^3	5	1
s^2	2	5
s^1		
s^0		

$b_2 =$

SS 23

s^0, s^2, s^4 all share s^0 constant in polynomial

Third Order Trick → 3rd order polynomial, if even 1 of the coefficients has a different sign, you will have RHP roots

SS 26

Examples:

1. YES
2. YES
3. $523s^2 - 57s + 189$

s^2	523	189
s^1	-57	0
s^0	189	

↳ NO, two roots in RHP

4. $s^4 + 2s^3 + s^2 - 1$

s^4	1	1	-1
s^3	2	0	
s^2	1		
s^1			
s^0			

Answers:

1. YES
2. YES
3. NO
4. NO
5. NO

Lecture 7

$$\lim_{\varepsilon \to 0^+} \frac{\frac{6 \cdot (4\varepsilon - 12) - 10\varepsilon}{\varepsilon}}{\frac{4\varepsilon - 12}{\varepsilon}} = \lim_{\varepsilon \to 0^+} \frac{24\varepsilon - 72 - 10\varepsilon^2}{4\varepsilon - 12} = \frac{-72}{-12} = 6$$

ROZ, only element in the row is zero

Auxillery Polynomial, roots of this polynomial can give you roots on $j\omega$ axis

$s^4 + s^3 + 3s^2 + 2s + 2$

$(s^2 + 2)(s^2 + s + 1)$

Polynomial → Routh Hurwitz Matrix

exponential / sinusoidal terms → Nyquist criterion

** Example 6 → This is a good exam / Quiz question

* All parameters / constants are assumed to be positive and not equal to zero

① $G(s)H(s)$ = Open loop Transfer function or loop gain

② $\frac{Y(s)}{X(s)}$ = Closed loop transfer function

* 1 + (Open loop transfer function) = 0 ⟷ Characteristic equation

↕ Find Poles = Stability

** Example 7 ⟶ Good Quiz / Exam question

$s^3 + 4s^2 + 5s + 2$

$(s+1)(s^2+3s+2)$

$(s+1)(s+1)(s+2)$

$= (s+1)^2(s+2)$

5/16/19
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Lecture 7 Cont'd

SS 26 | Example 8 → GOOD practice question

Auxillery Polynomial — Roots = poles on jw axis

$p'(t) = at$

↳ 'a' constant replaces the ROZ zero

Lecture 8 S.S. Error

SS 8

$$y(t) = \underbrace{y_t(t)}_{\text{transient}} + \underbrace{y_{ss}(t)}_{\text{steady-state}}$$

Transient → dies out $\lim_{t\to\infty} y_t(t) = 0$

Steady-state → Forced response, remains after

SS 12

Settling Time, 2% T_s → find 2% above and below y_{ss}. When it enters the box for the first time and STAYS within the box forever after, mark where it enters, that is your T_s

Steady state error: *(For input u(t))

$e_{ss} = 1 - y_{ss}$

General, e_{ss} = expected endpoint $- y_{ss}$

Percent Overshoot (PO):

$PO = \frac{y_{max} - y_{ss}}{y_{ss}} \cdot 100\%$ → "Indication of Stability of System"

$H(s) = \frac{k}{Ts+1}$ where $T =$ ** $\boxed{\text{Time Constant}}$ → 63% value

$\boxed{\begin{matrix} T_d = 0.7T \\ T_r = 2.2T \end{matrix}}$ First order only

Design Specifications ⟷ Performance Measures

L(s) = Open loop Transfer Function

→ = Forward Transfer function

① Step-error → $k_p = \lim_{s \to 0} L(s)$

② Ramp-error → $k_v = \lim_{s \to 0} s L(s)$

③ Parabolic-error → $k_a = \lim_{s \to 0} s^2 L(s)$

*want to be as big as possible to minimize e_{ss}

Lecture 8 Cont'd

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SS 23

Steady state Error:

① Step $Ru(t) \rightarrow e_{ss} = \frac{R}{1+k_p}$

② Ramp $Rtu(t) \rightarrow e_{ss} = \frac{R}{k_v}$

③ Parabolic $\frac{Rt^2}{2}u(t) \rightarrow e_{ss} = \frac{R}{k_a}$

SS 26

Accurate Tracking ($e_{ss} = 0$):

① Step $u(t) \rightarrow$ need s^1 or greater in denominator

② ramp $tu(t) \rightarrow$ need s^2 or greater in Denominator

③ Parabolic $\frac{t^2}{2}u(t) \rightarrow$ need s^3 or greater in denominator

SS 27

System Type

$$T(s) = \frac{K(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}$$

Type n system → "n defines system type"

Lecture 9

2nd Order:

$$G(s) = \frac{k}{s^2 + ks + k}$$ "k's don't have to be the same"

Standard Form:

"Divide numerator and denominator to obtain form" $\frac{k}{Ts+1}$

① $k \rightarrow$ "DC gain" $\rightarrow \lim_{t\to\infty} y(t) = k \rightarrow$ "Final Value"

② $T \rightarrow$ "63% of Final Value" $\rightarrow 0.63 \cdot k \rightarrow$ Time when this happens

How to find T?

initial slope $(t=0) = \frac{1}{T}$

Settling time: (For first order) → "no box"

2% → time when @ $0.98 \cdot$ Final Value
5% → time when @ $0.95 \cdot$ Final value

"Summarizes everything on 1st Order Systems"

Lecture 9 Cont'd

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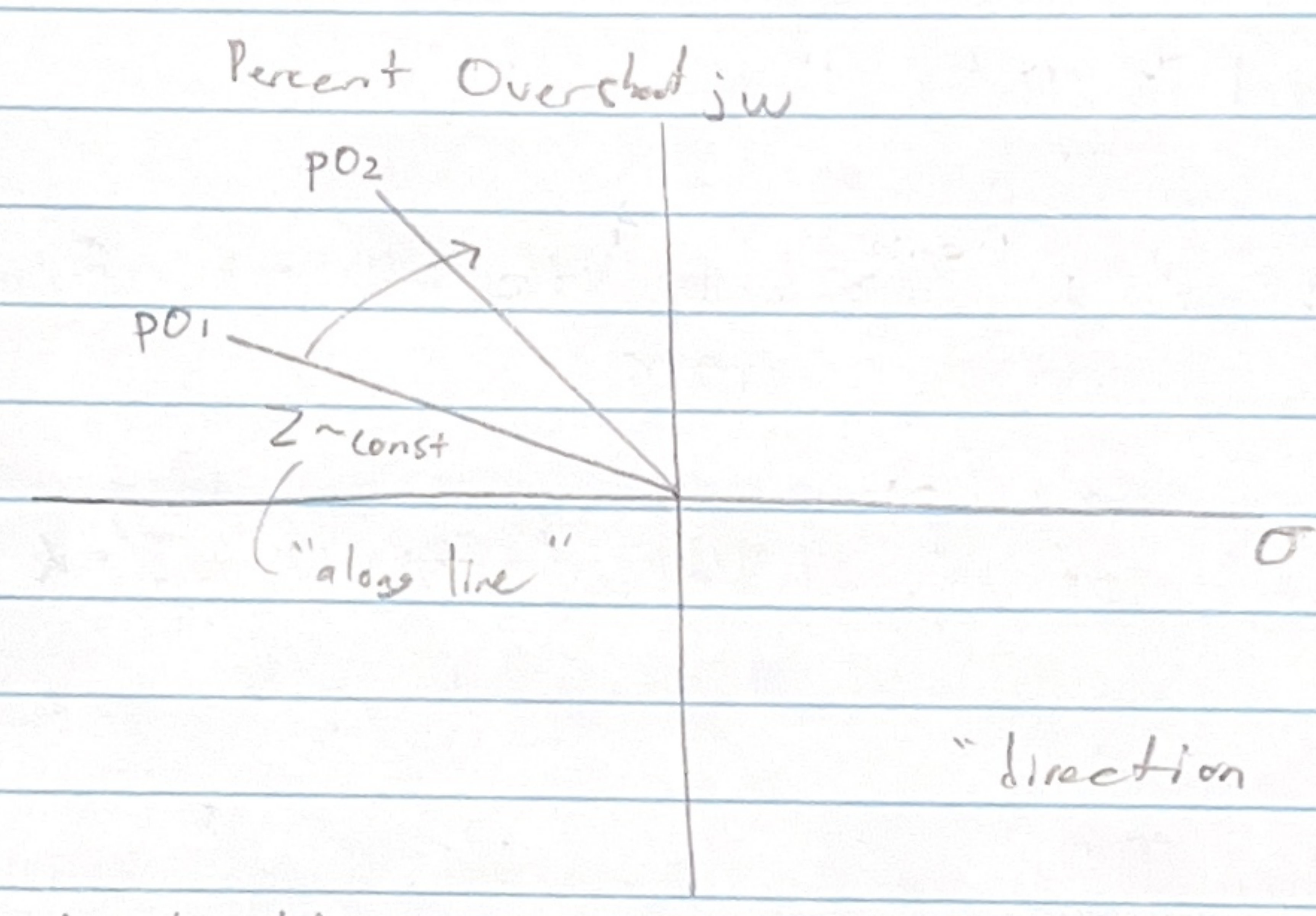
[SS 22] * Good slide for formulas

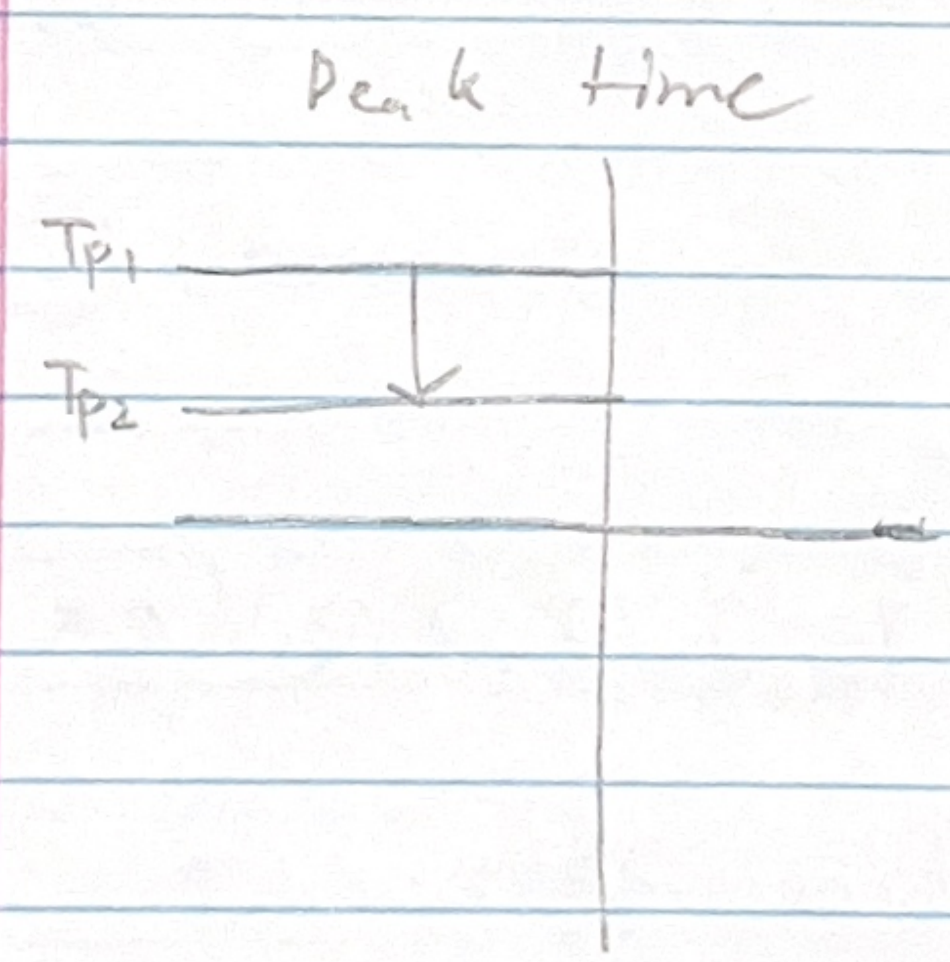
$\boxed{T = \frac{1}{Z \cdot \omega_n} = C}$ 2nd Order System

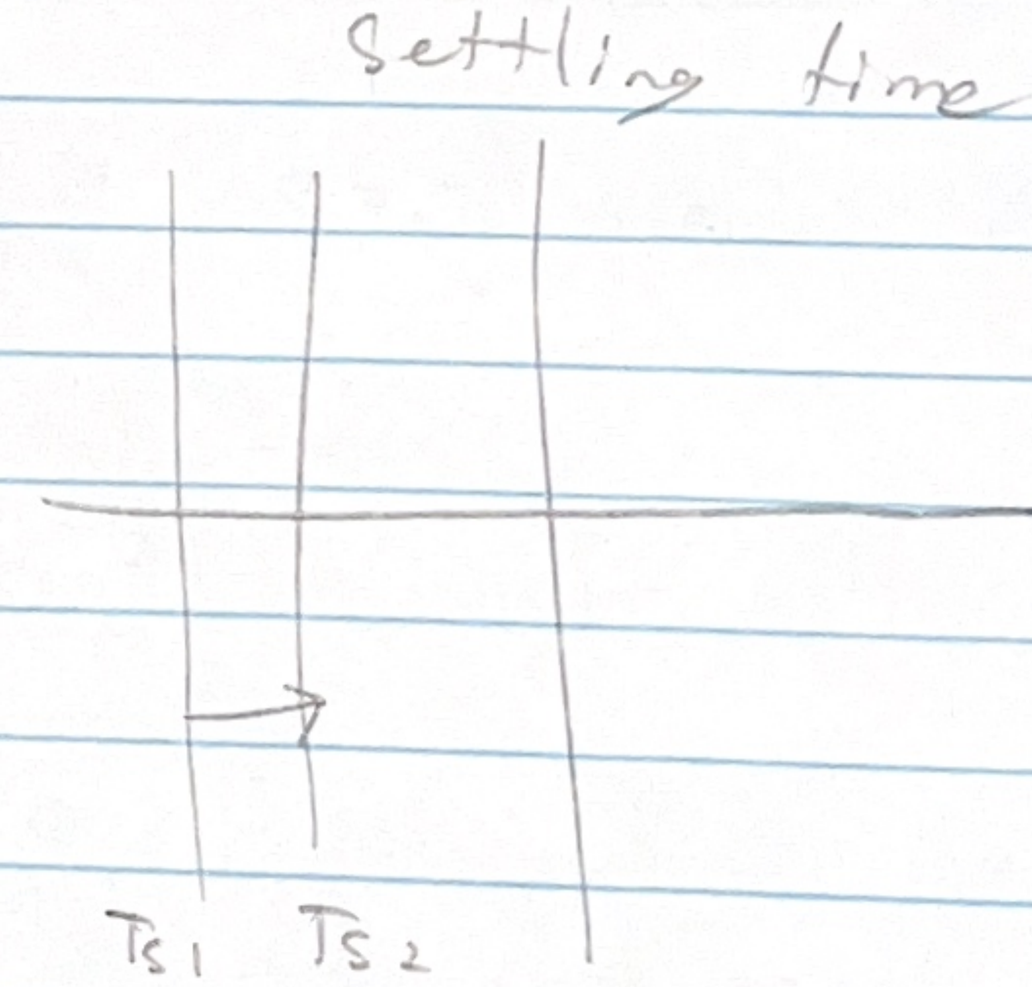
[SS 25] * location of a pole gives you everything

[SS 30]



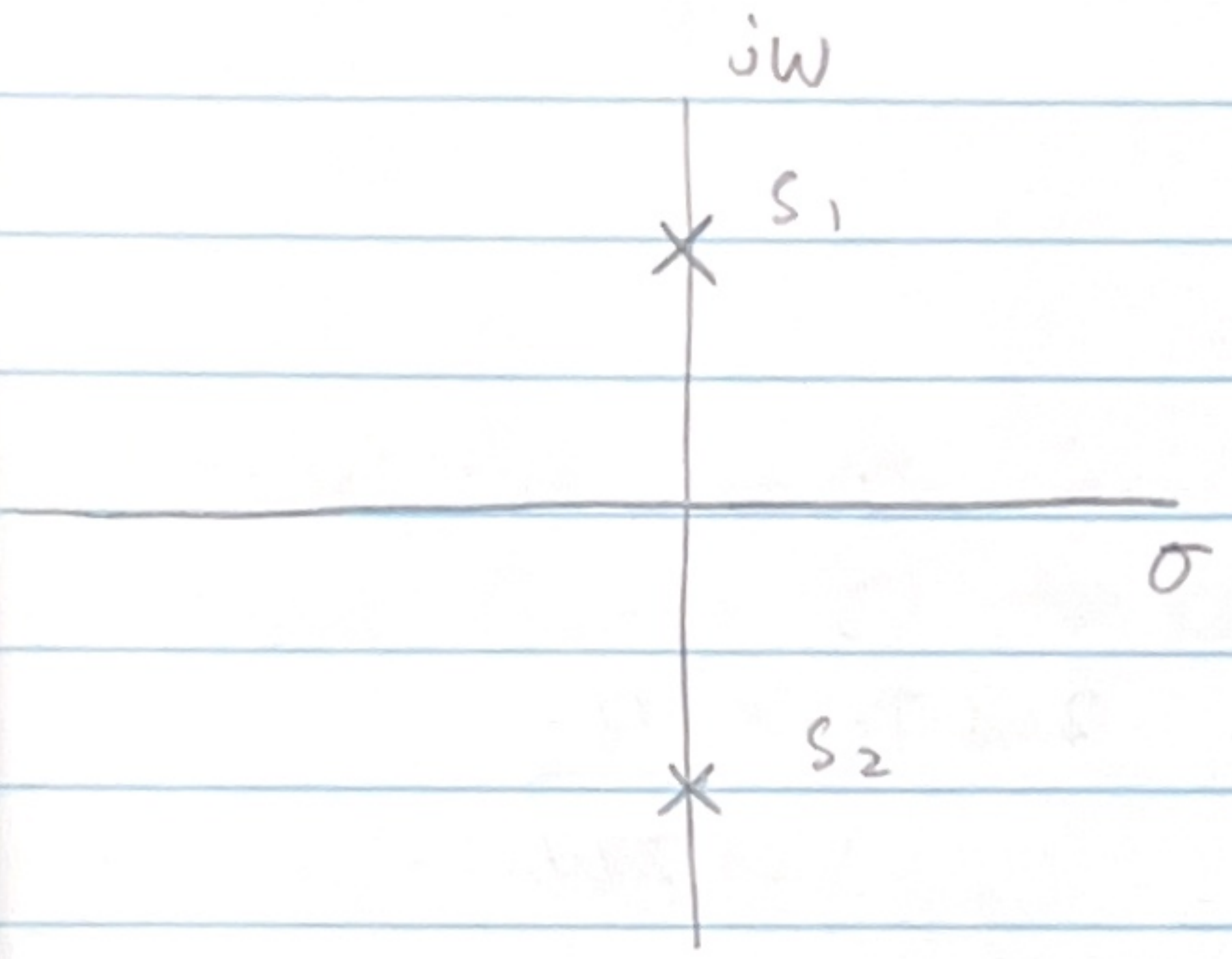
"direction of increase"





"What changes as we move pole location"

Lecture 10



"Find jw crossing"
"@ what frequency does your system oscillate"

$$S_{1,2} = \pm j\omega_n$$

① Undamped → Poles on jw axis

② Underdamped → Complex Poles

③ Critically damped → Double real poles

④ Overdamped → Two Distinct real Poles

Percent overshoot → NOT defined for first order systems "no oscillation"

Lecture 10 cont'd

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SS 30

$\frac{A}{s+2.5} \rightarrow 2\%\ T_s = 4T$

2^{nd} order $\rightarrow T = \frac{1}{Z\omega_n} \rightarrow 2\%\ T_s = \frac{4}{Z\omega_n}$

SS 31

"Not sufficiently dominant"
↳ Needs to be 5-10 times further away

** **SS 33**

ADD Formula to sheet.

SS 35

*Must be underdamped to use all the nice formulas

SS 36

$20\%\ T_s = 4T$
$5\%\ T_s = 3T$

SS 40

*solve for Practice

$K_v = \lim_{s \to 0} s \cdot (\text{Open-loop TF})$

Quiz 1
LEC 1-10

Routh-Array
Stability
Reduce Block diagram

A4 size Paper

Lecture 11

Root Locus → shows how poles of TF change as k goes from $0 \to 100$

Root-Locus sketching:

"often just graph top half → Symmetry"

# of branches = # of Poles of $L(s)$

0 Arrow always in direction:

Open loop Poles ⟶ Open loop Zeroes

12 Relative degree = degree(denominator) − degree(numerator)

13 $\frac{\Sigma \text{Pole} - \Sigma \text{zero}}{r}$ = Asymptote Location

Asymptote = - - - - - - - -
Root Locus = ⟶

15 ① $\frac{dL(s)}{ds} = 0$

② Find $s_{1,2,3}$ roots

③ $k = \frac{-1}{L(s)}$, Plug in $s_{1,2,3}$

④ $k > 0 \to s$ = breakaway Point

Lecture 13 Cont'd

$(s^2+4s+3)^{-1}$

$(-1)(s^2+4s+3)^{-2} \cdot (2s+4)$

$$\frac{(-1)(2s+4)}{(s^2+4s+3)^2}$$

$$\frac{(-1-3)-(0)}{2} = \boxed{-2}$$

When calculating error constants:

① Use the original $L(s)$
NOT the fictitious OLTF

$Z=1 \rightarrow$ Critically damped $\rightarrow$ Similar to 1st order system

$$T_s = \frac{4}{Z\omega_n} \leq 1$$

Increasing gain reduces ess BUT may increase settling time (ie: oscillation)

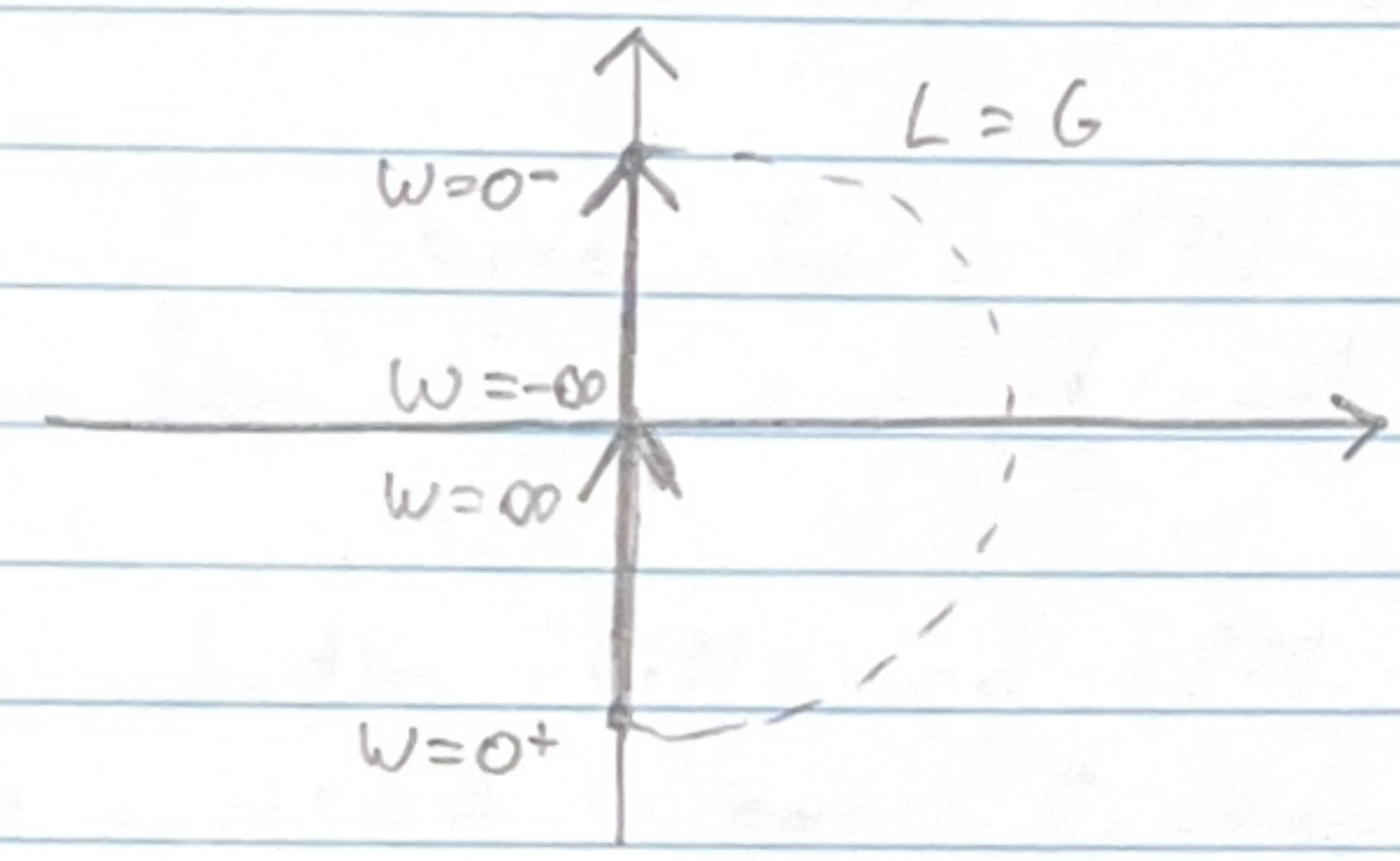
Standard Form For Root Locus:

$$1 + k \cdot L(s) = 0$$

Lecture 18

Nyquist plots Gain/Phase on the same plot

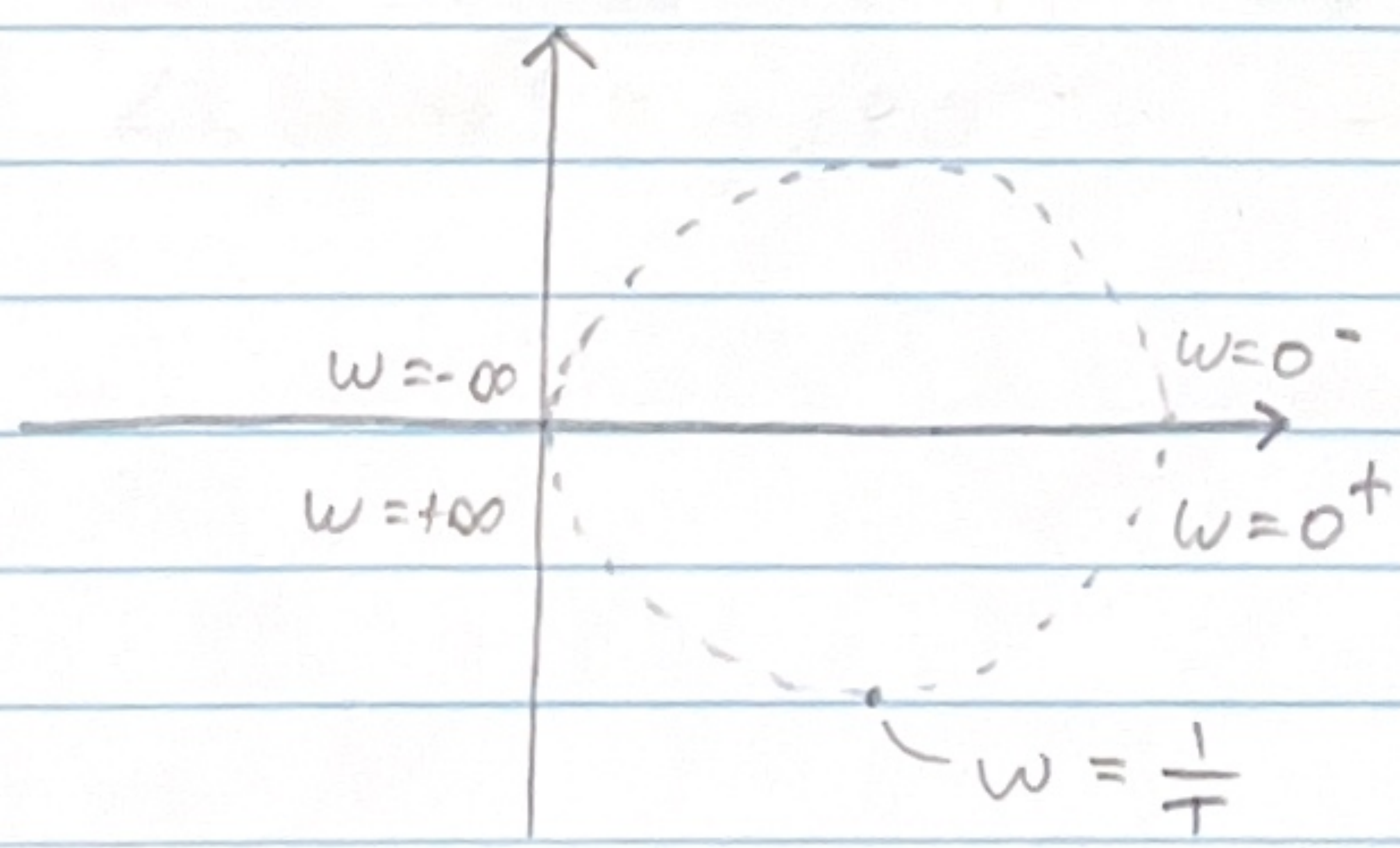
Arrow → is in the direction of increasing ω



*Trick: if zeros are separted connect them with the dotted half circle

Transfer Function: FRF (Frequency response Function):

$T(s) = \frac{1}{sT+1}$ $T(c) = \frac{1}{(j\omega)T+1}$



*Do not forget Direction Arrows

Z = 0 means stability for CL

Lecture 19 Cont'd

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SS 28 — If he says find GM/PM. Do it analytically <u>not</u> Graphically

(1) But, if he gives graphs (Easy) then you can do graphically

W_g intersects at 0DB

*Add to Formula Sheet → Graphical method

SS 29 — (1) e^{-s} Delay does not affect Bode magnitude plot
(2) It <u>does</u> affect phase plot

SS 30 — GAP → W_g left of W_p = STABLE
PAG → W_p left of W_g = UNSTABLE

*(Only up to slide 18 is examinable)

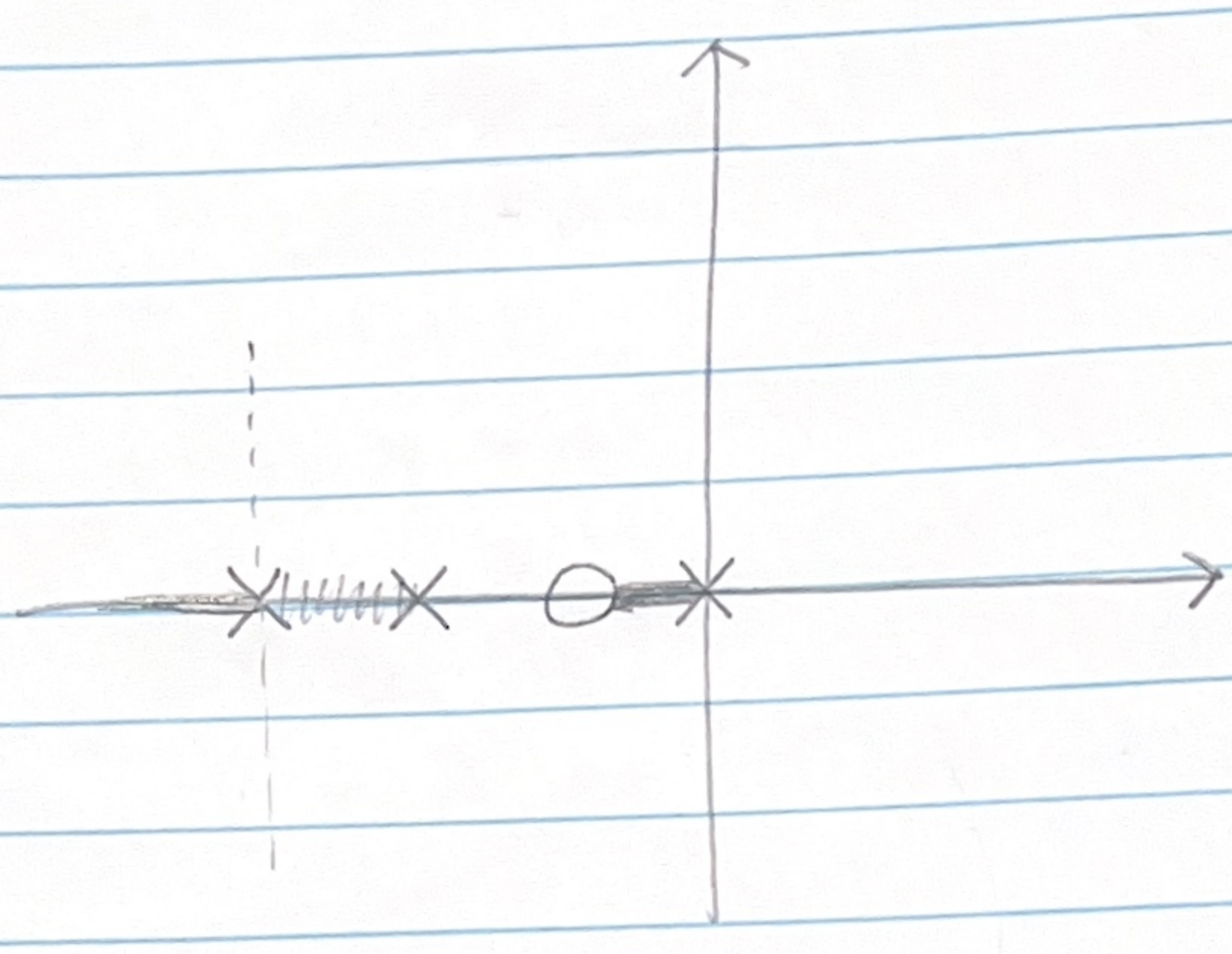
Lecture 20

SS 10 — $\frac{1}{(s+1)^2}$ — $W_p = \infty$ — GM = ∞

$W_g = 0$ — PM = 180

SS 11 — $\frac{1}{(s+1)^3}$ — $W_p = \sqrt{3}$ — GM = 18.06

$$L(s) = \frac{s+1}{s(s+2)(s+3)} \qquad r = 2$$



$$\alpha = \frac{[0-5]+[-1]}{2} = -3$$

$$\alpha = \frac{[\text{Pole}]-[\text{Zeros}]}{r}$$

① # of branches = order of $L(s)$

② # branches $\to \infty$ = relative order